Naturalness
&
Compositeness
2014

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In QM whatever is *possible* is also *compulsory*

**Selection rules**

\[ O = \sum_i O_i \]

\[ O_i = c_i \lambda_1^{n_{1i}} \cdots \lambda_k^{n_{ki}} \]

If \( O_{\exp} \ll \max |O_i| \) it seems we are missing something

Un-Naturalness = failure of dimensional analysis and selection rules
Mass Hierarchies

\[ \Lambda_{UV} \]

\[ \Lambda_{IR} \]

\[ \sim \text{ scale invariant dynamics} \]

\[ \mathcal{L} \sim \text{fixed point of RG} \]

Naturalness of \( \Lambda_{IR} \ll \Lambda_{UV} \) \leftrightarrow \text{stability of fixed point}

3 options
1. Marginality

The fixed point theory does not possess scalar operators with dimension strictly less than 4.

\[
\mathcal{L}_{\text{mass}} = c \Lambda_{UV}^\epsilon \mathcal{O}_{4-\epsilon}
\]

\[
\Lambda_{IR}^\epsilon = c \Lambda_{UV}^\epsilon
\]

\[
\Lambda_{IR} = c^{1/\epsilon} \Lambda_{UV}
\]

Algebraically small \( c \) and \( \epsilon \) is enough to produce hierarchy.


Ex: Yang-Mills, TechniColor, Randall-Sundrum model
2. Symmetry

\[ \Lambda_{UV} \quad \frac{\text{________________} \quad \rightarrow \quad \Lambda_{IR} \quad \frac{\text{________________}}{} \quad \Rightarrow \quad \mathcal{L}_{\text{mass}} = \epsilon \Lambda_{UV}^2 \mathcal{O}_2 \]

small parameter protected by symmetry

\[ \Lambda_{IR} = \sqrt{\epsilon} \Lambda_{UV} \]

- \( \epsilon \) must be \textit{hierarchically} small
- how does this smallness originate?

Ex: QCD, Supersymmetry
3. Sequestering

\[ \Lambda_{IR} \quad \Lambda_{UV} \]
3. Sequestering

$\Lambda_{IR}$

$\Lambda_{UV}$
3. Sequestering

$\mathcal{O}(\epsilon)$

$\Lambda_{IR}$

$\Lambda_{UV}$
3. Sequestering

\[ \Lambda_{IR} \sim \epsilon \Lambda_{UV} \] technically natural

- SFT2 = ‘UV completion of gravity’\[ \epsilon = \Lambda_{UV}/M_P \]
- not clearly compatible with basic principles
- but imagine we find a gorgeous candidate for SFT1?

Ex.

\[ a \text{-} gravity \]
\[ \text{by} \]
\[ a \text{-} strumia \]
The Standard Model

\[ \Lambda_{UV} \]

\[ \Lambda_{IR} \sim m_W \]

\[ L_{SM} \sim L_{free} \equiv \text{fixed point} \]

\[ H^\dagger H \equiv \text{relevant and unprotected} \]

but tuning comes with a bonus

\[ L_{SM} = L^{(d \leq 4)} + \frac{1}{\Lambda_{UV}}L^{(5)} + \frac{1}{\Lambda_{UV}^2}L^{(6)} + \ldots \]

- Accidentally possesses all the symmetries we observe in Nature: B, L, Flavor, ...
- Not the case in any natural completion of the SM
Composite Higgs Scenario

\[ \sim CFT \]

\[ h, W^{\pm}, Z^{L} \]

\[ q, \ell, \gamma, W_{T}, Z_{T}, g \]
Weak scale structure

$H = \begin{pmatrix} h_1 + ih_2 \\ h + ih_3 \end{pmatrix}$

must be a pseudo-Golstone multiplet

Ex.: $H \in SO(5)/SO(4)$

---

$W^\pm_L, Z_L, h$

---

$\sim CFT$

---

Georgi, Kaplan '84
Banks '84

Arkani-Hamed, Cohen, Katz, Nelson '02
Agashe, Contino, Pomarol '04
EWSB is *broadly* described by

✧ one mass scale $m_*$

✧ one coupling $g_*$

\[ g_* \sim m_* \]

Ex.: $g_* \sim \frac{4\pi}{\sqrt{N}}$

\[ g_* \bar{\Psi} \Psi \Phi \]

\[ \frac{g_*^2}{m_*^2} (\pi \partial \pi)^2 \]

$h \in \pi = \text{pseudo-NG}$

\[ \frac{g_*}{m_*} \equiv \frac{1}{f} \]
Flavor
The two ways to Flavor

**Bilinear:** ETC, conformal TC
- Dimopoulos, Susskind
- Holdom
- ....
- Luty, Okui

**Linear:** partial compositeness
- D.B. Kaplan
- ....
- Huber
- RS with bulk fermions
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**disfavored by CFT ‘theorems’**
- Rychkov, Rattazzi, Tonni, Vichi 2008
- Poland, Simmons-Duffin, Vichi 2011
Flavor from partial compositeness

\[ \mathcal{L}_{Yukawa} = \epsilon_q^i q_L^i \Psi_q^i + \epsilon_u^i u_L^i \Psi_u^i + \epsilon_d^i d_L^i \Psi_d^i \]

\[ H \]

\[ Y_{ij}^u \sim \epsilon_q^i \epsilon_u^j g* \]

\[ Y_{ij}^d \sim \epsilon_q^i \epsilon_d^j g* \]

\[ \Psi = \text{composite with dimension} \sim \frac{5}{2} \]

- Hypothesis seems a bit wishful, but no other option is in sight
- Problems of minimal technicolor greatly alleviated, but not eliminated
Flavor transitions controlled by selection rules

\[ \Delta F=2 \]

\[ \Delta F=1 \]

\[ \epsilon_q^i \epsilon_d^j \epsilon_q^k \epsilon_d^\ell \times \frac{g^2}{m^2} \left( \bar{q}^i \gamma^\mu d^j \right) \left( \bar{q}^l \gamma_\mu d^\ell \right) \]

\[ \epsilon_q^i \epsilon_u^j g_* \times \frac{v}{m^2} \times \frac{g^2}{16\pi^2} \bar{q}^i \sigma_{\mu\nu} u^j G_{\mu\nu} \]
### Flavor and CP bounds

<table>
<thead>
<tr>
<th>$\Delta F=2$</th>
<th>$\Delta F=1$</th>
<th>edms</th>
<th>$\mu \rightarrow e\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\epsilon_K, \ldots)$</td>
<td>$(\Delta c_{CP}^D, \epsilon'/\epsilon, b \rightarrow s\gamma)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_* &gt; 15$ (TeV)</td>
<td>$g* / 4\pi \times (10 - 15)$</td>
<td>$g* / 4\pi \times (50 - 200)$</td>
<td>$g* / 4\pi \times 200$</td>
</tr>
</tbody>
</table>

Partial compositeness is likely not the full story
Flavor and CP symmetry must be assumed

Range of possibilities

$U(1)_e \times U(1)_\mu \times (1)_\tau$

... ... ...

$SU(3) \times SU(3) \times ...$

Redi, Weiler ’11
Barbieri et al. ’12
The most clever set up

\[ SU(3)_{comp} \times SU(3)_Q \times SU(3)_D \times SU(2)_U \]

\[ \hat{Y}_D = \hat{\lambda}_D \epsilon_D \]

\[ \hat{Y}_U = \hat{\lambda}_U \epsilon_U + \hat{\lambda}_t \epsilon_{t_R} \]

**\( \epsilon_{U,D} \)**
- sufficiently small to satisfy bounds from light quark compositeness
- sufficiently large to avoid sizeable flavor violation from \( \hat{\lambda}_{U,D} \)

**\( \epsilon_{t_R} \)**
- sufficiently large to comfortably account for top Yukawa

\[ y_t = |\hat{\lambda}_t| \epsilon_{t_R} \]
CP conserving strong dynamics \rightarrow \text{phase alignment controls edms}

\[ \epsilon_U \sim 0.2 - 0.5 \rightarrow \text{constraint from compositeness and } \hat{\lambda}_U \text{ subdominant} \]

Uneliminable effect via the top doublet

\[ m_* g_* > \frac{5 \text{ TeV}}{\epsilon^2_{t_R}} \geq 5 \text{ TeV} \]
Higgs’s mass versus top-partners’

\[ V(h) = t_L T \times g^2 \epsilon_t^{2} + t_R T \times g^2 \epsilon_t^{2} + \ldots \]

\[ \propto g_*^2 \epsilon_t^{2} \]

\[ y_t \sim \epsilon_{t_L} \epsilon_{t_R} g_* \]

best option \( t_R \) is fully composite SO(5) singlet

\[ \epsilon_{t_R} = 1 \]
\[ \epsilon_{t_L} g_* = y_t \]

\[ V(h) = \frac{m_*^4}{g_*^2} \times \frac{y_t^2}{16\pi^2} \times F(h/f) \]

Mrazek et al, ’11
Panico, Wulzer ’11
Pomarol, Riva ’12
Higgs’s mass versus top-partners’

\[ V(h) = \alpha g_*^2 \epsilon_{t_L}^2 + \alpha g_*^2 \epsilon_{t_R}^2 + \ldots \]

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Pomarol, Riva ’12
The connection between $g_*, m_*, m_t$ and $m_h$

\[
V = \frac{3y_t^2 m_*^2}{16\pi^2} \left( a h^2 + b h^4 / f^2 + \ldots \right)
\]

\[
\xi \equiv \frac{v^2}{f^2} = \frac{a}{b}
\]

\[
m_h^2 = b \frac{3g_*^2}{2\pi^2} m_t^2 \sim (125 \text{ GeV})^2 \frac{g_*^2 b}{4}
\]

Total tuning $\sim$ area $= a b = \left( \frac{430 \text{ GeV}}{m_*} \right)^2 \times \frac{4}{g_*^2}$

De Simone, Matsedonskyi, RR, Wulzer 2012
Notice impact of 125 GeV Higgs

\[ m_h = 125 \text{ GeV} \quad \Rightarrow \quad a b = \left( \frac{430 \text{ GeV}}{m_*} \right)^2 \times \frac{4}{g_*^2} \]

weakly strong EWSB sector and light resonances preferred

\[ m_h = 250 \text{ GeV} \quad \Rightarrow \quad a b = \left( \frac{860 \text{ GeV}}{m_*} \right)^2 \times \frac{16}{g_*^2} \]

moderately strong and heavy EWSB sector
Higgs couplings

\[ a \times \frac{2m_V^2}{v} \]

\[ b = a^2 = 1 - \frac{v^2}{f^2} \equiv 1 - \xi < 1 \]

robust consequence of coset structure

\[ c_i \times \frac{m_i}{v} \]

\[ c_i \simeq 1 + O\left(\frac{v^2}{f^2}\right) < 1 \]

generic but not theorem

No other parameter at leading order in \( g_{SM}^2 / g_\ast^2 \)
\[ \Delta \epsilon_3 = O(1) \times \frac{m_W^2}{m_*^2} + \frac{g^2}{96\pi^2} \frac{v^2}{f^2} \ln\left(\frac{m_*}{m_h}\right) + \frac{3g^2}{32\pi^2} \frac{\tan \theta_W^2}{f^2} \ln\left(\frac{m_*}{m_h}\right) \]

\[ m_* \gtrsim 2 \text{ TeV} \]

\[ \Delta \epsilon_1 = \delta \rho_{SM} \times \frac{m_t^2}{m_*^2} - \frac{3g^2}{32\pi^2} \frac{\tan \theta_W^2}{f^2} \ln\left(\frac{m_*}{m_h}\right) \]

in principle very strong bound: \( \xi \equiv \frac{v^2}{f^2} \lesssim 0.05 \)

in practice it could be relaxed by short distance contribution
Figure 7

p Left: Probability distribution for the coupling $a$
Center: Indirect determinations of the coupling $a$, excluding the observables $M^\nu W$, $\Gamma_Z$, $P^{Pol}_\tau$, $A_I$, and $A_{0,b}^{FB}$, except for the one specified in each row.
Right: Probability regions in the $a$–$\Lambda$ plane. In all plots, the large heavy expansion is adopted to the two-loop fermionic EW corrections to $\rho$.

3.5 General bounds on the New Physics scale

Before concluding, let us take a more general approach and consider the contributions to the EW fit of arbitrary dimension six New Physics induced operators $O_{ijkl}^a$:

\[
L_{eff} = L_{SM} + \sum_i C_i \Lambda^2 O_i^a.
\]

For concreteness, let us use the same operator basis of ref $ss$: $O_{WB}$, $O_{H}$, $O_{LL}$, $O_{\rho L}$, $O_{HQ L}$, $O_{\rho U}$, $O_{HQ U}$, $O_{HE}$, $O_{HD}$.

The first two operators contribute to the oblique parameters $S$ and $T$:

\[
S = \frac{v}{\sqrt{2}} \frac{s_w^2 W^2 C_{WB}}{\Lambda},
\]

\[
T = -\frac{C_{H_u}}{\sqrt{2}} \frac{M^2 \Lambda}{s_w^2 W^2 C_{WB}}.
\]

Where $O_H$ violates the custodial symmetry, since it gives a correction to the mass of the $Z$ boson, but not to that of the $W$ boson. The next two operators yield non-oblique.

Franco, Mishima, Silvestrini 2013
Direct searches (LHC 8 TeV)

- **Top partners** \((Q = -\frac{1}{3}, \frac{2}{3}, \frac{5}{3})\) \(m_* \gtrsim 1\) TeV

- **Vector resonances**

\[ V = g_* V W_L \]

\[ m_* > 3\) TeV \]

\[ m_* > 2\) TeV \]

CMS data

Pappadopulo, Thamm, Torre, Wulzer 2014

\( g_* = 1 \)

\( g_* = 3 \)
\[ m_*^2 \xi = g_*^2 v^2 \]
$m_*^2 \xi = g_*^2 v^2$

Higgs couplings

direct searches
A. Thamm 2014

Top partners LHC8
In my opinion

- Compositeness remains a comparatively viable option to solve the hierarchy problem.

- It also forces us to think more and better about QFT

- Flavor is its major structural drawback

- Flavor and EWPT make the outcome of LHC8 unsurprising

- LHC13 and HL-LHC will definitely break new grounds